

Progression In Calculations at Homefield Primary School and SSC



Addition and Subtraction

Mathematical Calculations in School Today.

This document is designed to help you to understand the calculation methods your child will be taught in school. When supporting your child at home with Maths work it would be helpful if you could reinforce these methods rather than teach them the way that you were taught. Please speak to your child's teacher to find out which methods would currently be the most appropriate for your child to practice at home.

Remember each child will progress at their own pace.



Understanding Addition and Subtraction

Explanation

The physical act of counting out a set number of objects, and combining two groups or taking some away, is an important step for children to explore. This is best done in a practical play based context as much as possible.

*Use a Numicon shape and add on the 1 shape...
What number do you have now? Find the new
Numicon shape to cover over the top.*

*Count out a given number of objects and take away 1
of them... How many do you have now?*

Understanding addition as combining two groups

Children need to experience counting out a set of objects and combining them with another set of objects to make a total amount. Initially this needs to be adding 1 more to numbers up to 10, then 20.

Explore 1 more through simple songs and rhymes, for example '1 man went to mo' or '1, 2, 3, 4, 5, once I caught a fish alive'.

Understanding subtraction as taking away

Children need to experience counting out a set number of objects and then removing/taking away a certain number from that group. Initially this needs to be taking away 1 from a set of up to 10 and then 20.

Explore 1 less through simple songs and rhymes, for example '10 green bottles' or '5 little speckled frogs'.

Key Questions/Vocabulary

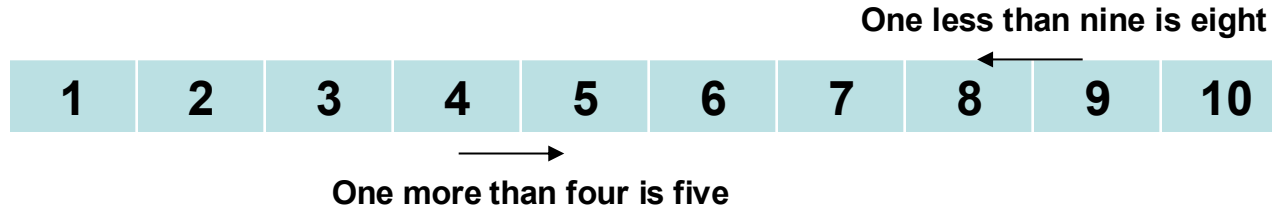
More, more than, one more, after, add, plus, count, total, equals, makes
Less, less than, one less, before, take away, subtract, leaves,
What is the number after 6? What is the number before 9?
How will you find out how many there are in total/left?
Can you show me how you worked out 1 more than/1 less than.....?

Progression

- Start by adding and taking away 1 from a number up to 10, then up to 20
- Progress to adding/taking away more than 1 within the same number ranges



Using a Number Track for Addition and Subtraction



Explanation

Number tracks can be used to help a child locate a number in a sequence, learn the order of numbers, and begin to find one more or one less than a given number. Once they understand the direction of movement for addition and subtraction they can progress to finding more than 1 more or less

Children need to be able to understand the order of numbers remains the same and that as we count on the numbers get bigger by 1 and as we count back the numbers get smaller by 1. They need plenty of practise in counting objects and by rote.

Counting on: Count the stairs as you go up to bed... Count your footsteps as you walk across a room.... Write the numbers to 10 on separate pieces of paper and get your child to put them into order, counting to check., then progress to ordering numbers to 20.

Counting back: Count back as you go down the stairs.... Do a countdown from 10 or 20 before you leave the house.... Write the numbers to 10 on separate pieces of paper and get your child to order them in reverse, then try from 20.

Key Questions/Vocabulary

Count on, add, more, one more, more than, equals, totals, makes

Count back, take away, subtract, one less, less than, leaves

Find the number that is *one more/less than*...?

Find the number that is *4 more/less than*...?

Count on/back 3 places from 15, where do you land?

How many is 5 more/less than 8?

*Order Numicon
1 – 10 set, place a
shape on top of the
next one, noticing
how there is a
difference of 1.*

Begin to know + and – facts within 5.

Begin to recall pairs of numbers that total 10.

Begin to know doubles up to double 5.

Progression

- Start by adding and taking away 1 from a number up to 10, then up to 20
- Progress to adding/taking away more than 1 within the same number ranges



Simple Jottings/Mark Making for Addition

$$13 + 2 = 15$$

Begin to know + and – facts within 5.
Begin to recall pairs of numbers that total 10.
Know doubles up to double 5.

Key Questions/Vocabulary

Count, count on, more, add, plus, sum, altogether, total, equals

How many altogether?

Find the number that is *five more than*...?

Count on 6 more from 13, what number do you get to?

Explanation

Simple mark making is the first stage of children's independent jottings to help them solve additions. They draw or make the appropriate number of marks under each number then count them up to reach the total. It is not necessary to draw the number of marks under the answer. Children can also use objects, such as counters, sweets, beads or cubes to create groups to combine to find the total of an addition.

Children need to begin to see that addition produces the same answer which ever way round it is solved eg $8 + 2 = 10$ and $2 + 8 = 10$. It is COMMUTATIVE. Lay the 8 and 2 Numicon shapes together and place another 2 then 8 on top to see it is the same. Encourage children to see that it is easier and quicker to count on from the largest number.

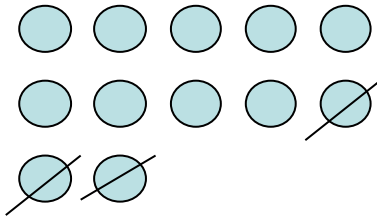
Progression

- Initially work with numbers up to 20
- Once children are confident counting beyond 20 and understand the place value of the digits in a 2-digit number, more efficient methods can be taught



Simple Jottings/Mark Making for Subtraction

$$12 - 3 = 9$$



Explanation

Simple mark making is the first stage of children's independent jottings to help them solve subtractions. They draw the initial number of objects and then cross off the number it says to take away and count the ones left over. Children can also use objects, such as counters, sweets, beads to create the initial group and then physically take away the right number to find the answer to the subtraction.

Begin to know + and – facts within 5.
Begin to recall pairs of numbers that total 10.
Know doubles up to double 5.

Key Question/Vocabulary

Count, count back, subtract, take away, cross off
Difference between, leaves, equals
How many are left over?
Find the number that is *6 less than...*?
Count back 5 from 16, what number do you get to?

At this level, children need to see that when doing subtraction the biggest number needs to be first and you take away the smaller number.

Progression

- Initially work with numbers up to 20
- Once children are confident counting beyond 20 and understand the place value of the digits in a 2-digit number, more efficient methods can be taught

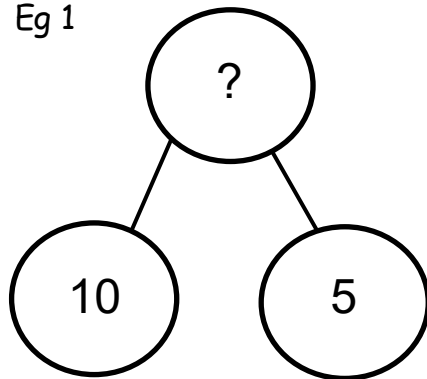


Part-Part-Whole Models

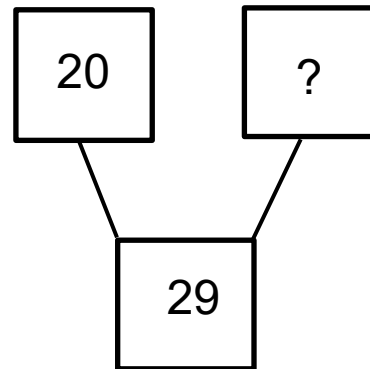
Explanation

A part-part-whole model shows the relationship between the whole number and its parts, and is a clear way for children to see the relationship between addition and subtraction. It is important for the models to be represented in different ways as shown in these examples. Try to get children to recall known facts to support their thinking. Ensure conceptual variation by looking at part-part-whole models alongside number lines and using known facts.

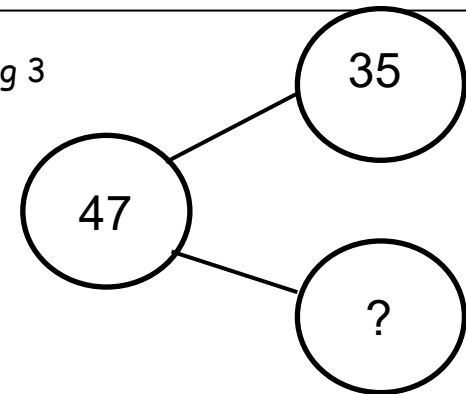
Eg 1



Eg 2



Eg 3



Key Questions/Vocabulary

Part, whole, combine, add, plus, subtract, take away, minus, equals, makes, total, altogether
What is this part? What is the whole?
What do I add to this part to make this whole?
If this is the whole and this is a part, what is the other part?

**Know addition facts for numbers within 10.
Rapidly recall all pairs of numbers that have a total of 10.
Begin to know doubles up to double 10 and corresponding halves.**

Progression

- To understand the model, show both parts completed and children have to combine them to find the whole. Begin with tens and units splits (Eg1)
- Progress to tens and unit splits where a part is missing to make the link to subtraction (Eg2)
- Move on to any 2 digit number as the whole or related parts (Eg3)



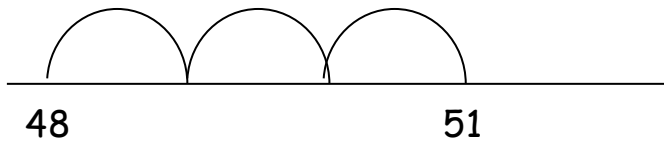
Using a Blank Number Line for Addition and Subtraction

Explanation

Blank number lines are used to enable children to count on and back with more than one jump. Children are taught to draw their own number line and start with the biggest number on the left for addition and the right for subtraction. There is no need to write +1 or -1 in each jump. Children learn to use ones jumps, adding and taking away single digit numbers. **This method is useful for supporting counting through tens boundaries.** It is only necessary to record where they start and where they end up after adding/subtracting. Remember to jump on/back from the biggest number! Ensure conceptual variation by looking at number lines alongside part-part-whole models and using known facts.

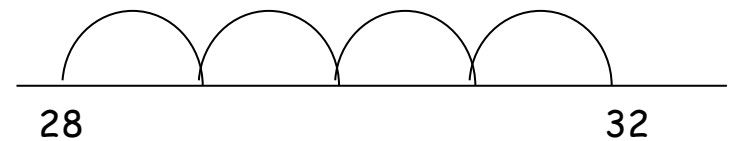
$$48 + 3 = 51$$

Jump this way



$$32 - 4 = 28$$

Jump this way



Know addition facts for numbers within 10.

Rapidly recall all pairs of numbers that have a total of 10.

Begin to know doubles up to double 10 and corresponding halves.

Key Questions/Vocabulary

Count on, count on in ones, add, plus, more than, total,

Count back, count back in ones, less than, take away,

Subtract, leaves, equals, makes

Which number are you starting with?

How many ones jumps do you need to do? Which direction?

What number have you reached?

Children can use Deines (one blocks) to place in the jumps and support the visual image of how many they need to add on/take away.

Progression

- Add and subtract single digit numbers within a range up to 100 (ensure it is within the child's counting range)
- Do not expect children to use this method with +/- two 2-digit numbers as this would become inefficient
- This might be used alongside adding/subtracting tens then ones by partitioning to support counting through the tens boundaries



Developing Jotting for Addition and Subtraction

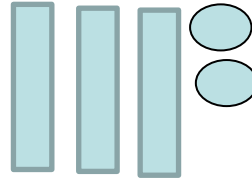
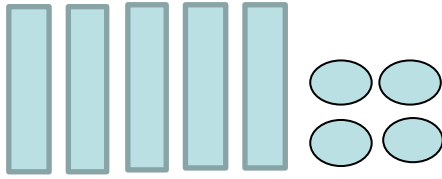
54

+

32

=

86



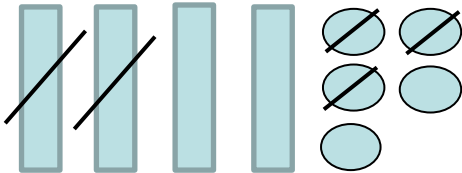
45

-

23

=

22



Sometimes these jottings are called '**Chips and peas**' to help support the drawing of them

Explanation

Once children have a secure understanding of place value, they can use this knowledge to support their calculations through simple jottings. We would use this method with the addition and subtraction of two 2-digit numbers that do not cross a tens boundary. For addition, draw the tens and ones for each number, then count them all. For subtraction, draw the tens and ones for the start number, then cross off the value of the digits in the subtracting number.

Key Questions/Vocabulary

Tens, ones, units

Count, count on, more, add, plus, sum, altogether, total, equals

Subtract, take away, cross off, leaves

How many altogether? How many are left?

Know addition facts for numbers within 10.

Rapidly recall all pairs of numbers that have a total of 10.

Begin to know doubles up to double 10 and corresponding halves

Progression

- Initially work with numbers that do not cross a tens boundary.
- Addition will work where a tens boundary is crossed (eg $46 + 37$) as the child can count on
- Subtraction will not work crossing a tens boundary



Using Known Facts for Addition and Subtraction

Explanation

Children need to be able to apply the facts that they know to support more efficient calculating. It is important that procedural variation is used frequently to help children make connections and see patterns with the facts that they know and other calculations. Ensure conceptual variation by looking at known facts alongside part-part-whole models and number lines.

$$63 + 6 = ?$$

$$\begin{aligned} 3 + 6 &= 9 \\ 13 + 6 &= 19 \\ 23 + 6 &= 29 \\ 33 + 6 &= 39 \\ 43 + 6 &= 49 \\ 53 + 6 &= 59 \end{aligned}$$

$$8 + 9 = ?$$

"I know that double 8 is 16 so 1 more makes it 17"

$$78 - 4 = ?$$

"I know that 8 minus 4 is 4 because double 4 is 8. So $78 - 4$ must be 74"

$$54 + 6 = ?$$

"I know that $4 + 6$ is 10 so $54 + 6$ must be the next tens number, 60"

Key Questions/Vocabulary

Double, half, near double, pairs to 10,
Tens, units, ones, add, plus, subtract,
minus, take away, total, makes,
equals

Do you know a fact that could help you?
Is it a near double?

Recall all + and – facts for each number within 10 and use to reason bonds to and within 20. Know doubles up to double 10 and their corresponding halves.

Progression

- Encourage children to apply their knowledge of doubles and halves as well as pairs to 10
- Progress to applying other known addition and subtraction facts as well as near doubles



Add/Subtract Tens Then Ones By Partitioning

Explanation

When we begin to cross boundaries with addition and subtraction, we use partitioning to break it down into steps and recording as shown. An arrow is included to make the connection that when we are adding on or taking away the units it is a continuation of the prior line of recording and the units are added on/taken away from the number we have already reached. Some children may need to use the blank number line method to support counting through the tens boundary.

To add 58 and 35....First partition the 2nd number into tens and ones. Then add the number of tens and record the answer. Circle this answer and draw an arrow to carry this number down to start the next row. Add the number of units to get the final total.

$$\begin{array}{r} 58 + 35 = \\ \quad \quad \begin{array}{l} 30 \quad 5 \end{array} \\ 58 + 30 = \textcircled{88} \\ \swarrow \\ 88 + 5 = 93 \end{array}$$

Key Questions/Vocabulary

Tens digit, ones digit, units
Partition, split, recombine
How many tens? How many ones?
How many altogether?

Recall all + and – facts for each number within 10 and use to reason bonds to and within 20. Know doubles up to double 10 and their corresponding halves.

$$\begin{array}{r} 81 - 46 = \\ \quad \quad \begin{array}{l} 40 \quad 6 \end{array} \\ 81 - 40 = \textcircled{41} \\ \swarrow \\ 41 - 6 = 35 \end{array}$$

To subtract 46 from 81....First partition the subtracting number into tens and ones. Then subtract the number of tens and record the answer. Circle this answer and draw an arrow to carry this number down to start the next row. Subtract the number on units to get the final answer

Progression

- Begin with calculations where the units do not cross the tens boundary
- Progress to calculations where the units do cross the tens boundary



Partition and Recombine for Addition

Explanation

Once children are secure with place value, some find this strategy a quick and easy method for addition, that they are soon able to do mentally. It is important to follow the progression guidance below for understanding this method and how to record it. Some children will be able to do this method for additions that do not cross the tens boundary but will find other methods easier when the calculations do cross the tens boundary.

To add 23 and 35....First add the number of tens, so
 $20 + 30 = 50$

Then add the number of ones, so $3 + 5 = 8$

Finally combine the answers to give the total,

so $50 + 8 = 58$

$$\begin{array}{r} 23 \\ + 35 \\ \hline 50 \\ + 8 \\ \hline 58 \end{array}$$

Key Questions/Vocabulary

Tens digit, ones digit, units
Partition, split, recombine
How many tens? How many ones?
How many altogether?

Recall all + and - facts for each number within 10 and use to reason bonds to and within 20. Know doubles up to double 10 and their corresponding halves.

$$\begin{array}{r} 76 \\ + 47 \\ \hline 110 \\ + 13 \\ \hline 123 \end{array}$$

The same method applies if you are adding two 2-digit numbers

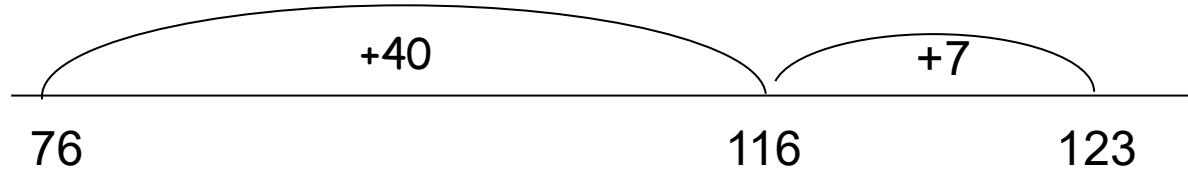
Progression

- Begin with calculations where the units do not cross the tens boundary
- Progress to calculations where the units do cross the tens boundary (eg $37 + 26$.
So $30 + 20 = 50$
 $7 + 6 = 13$
Then $50 + 13 = 63$)

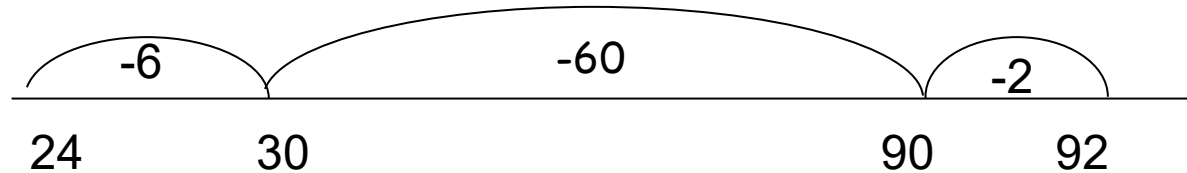


Develop Efficient Use of Number Lines

$$76 + 47 = 123$$



$$92 - 68 = 24$$



Explanation

Once children are confident and accurate in the use of tens and ones jumps, they can progress to using multiple of tens jumps. Encourage children to use their knowledge of number bonds to bridge to the nearest multiple of 10 to make counting easier (as in the second eg). Make sure they keep a record in their jumps of what they are doing so that they can check they have + or - the correct number.

Key Questions/Vocabulary

Addition, add, plus, more, more than
Subtract, take away, minus, less than
Ten more, ten less, count in tens
One more, one less, count in ones
Difference between, inverse
Equals, leaves, altogether

Know related subtraction facts for numbers within 10.
Know doubles up to double 10 and their corresponding halves.

Progression

- When looking at the number being added on or subtracted, begin by doing 'chunky jumps' of tens and then single units jumps
- Progress to bridging through tens boundaries with 'chunky jumps' of the units
- Generally in KS1, calculations remain within 100



Expanded Column for Addition and Subtraction

$$\begin{array}{r}
 76 \\
 + 47 \\
 \hline
 13 \quad (6+7) \\
 110 \quad (40+70) \\
 \hline
 123
 \end{array}$$

$$76 + 47 = 123$$

$$\begin{array}{r}
 276 \\
 + 147 \\
 \hline
 13 \quad (6+7) \\
 110 \quad (70+40) \\
 300 \quad (200+100) \\
 \hline
 423
 \end{array}$$

$$276 + 147 = 423$$

$$\begin{array}{r}
 \cancel{6}7\cancel{6}^{16} \\
 - 47 \\
 \hline
 9 \quad (16-7=9) \\
 20 \quad (60-40=20) \\
 \hline
 29
 \end{array}$$

$$76 - 47 = 29$$

If I subtract 7 from 6,
I get a negative number - I can't use this
within a calculation!
I need to exchange a ten
into ten ones: $10 + 6 = 16$.
 $16 - 7 = 9$

But now I can't subtract 70 from 60!
I need to exchange a hundred
into ten tens: $100 + 60 = 160$.
 $160 - 70 = 90$

$$\begin{array}{r}
 \cancel{2}\cancel{7}\cancel{6}^{16} \\
 - 177 \\
 \hline
 9 \quad (16-7=9) \\
 90 \quad (160-70=90) \\
 100 \quad (200-100=100) \\
 \hline
 199
 \end{array}$$

$$376 - 177 = 199$$

Explanation

Children then progress onto using an expanded column method. This method helps to prepare them for using the formal written method for addition and subtraction and builds on their knowledge of partitioning numbers into ones, tens and hundreds. They begin by calculating with the ones, before moving onto the tens and then the hundreds. They jot the calculation next to each row to keep track of each individual calculation. Once each column has been added or subtracted, children then total each column. When exchanging, children should write the digits above the relevant column. In most cases regrouping and exchanging will involve writing a 1 to represent a ten, however at times a different digit will need to be written where there has been an exchange as shown in the examples above.

Key Questions/Vocabulary

Addition, add, plus, more, more than
Subtract, take away, minus, less than
Difference between, inverse
Equals, leaves, altogether
Addend, sum, minuend, subtrahend
Regrouping
Exchange

**Know related
subtraction facts for
numbers within 20.**

Progression

- Children will first use this to add two 2-digit numbers. They will then calculate with 3 and 4-digit numbers.
- Once confident with subtraction, children will then calculate using exchanges.
- Once secure on this expanded method, they will progress onto a compact method.



Compact Column for Addition and Subtraction – Formal Written Method

$$\begin{array}{r}
 276 \\
 + 147 \\
 \hline
 423 \\
 \hline
 1 \quad 1
 \end{array}$$

$6 + 7 = 13$
 (Write the ten under the place value column to the left.)
 $7 + 4 + 1 = 12$
 (Write the ten under the place value column to the left.)
 $2 + 1 + 1 = 4$

$$276 + 147 = 423$$

$$\begin{array}{r}
 2 \quad 16 \quad 1 \\
 \cancel{3} \quad \cancel{7} \quad 6 \\
 - 177 \\
 \hline
 199 \\
 \hline
 \end{array}$$

If I subtract 7 from 6,
 I get a negative number – I can't use this
 within a calculation!
 I need to exchange a ten
 into ten ones: $10 + 6 = 16$.
 $16 - 7 = 9$

But now I can't subtract 70 from 60!
 I need to exchange a hundred
 into ten tens: $100 + 60 = 160$.
 $160 - 70 = 90$

$$376 - 177 = 199$$

Explanation

Once children are confident with calculating one column at a time, they then move onto the compact method. Children continue to calculate from right to left, starting with the ones column, however they now can only record digits in the answer space for the specific column they are working on and now begin to regroup (carry) tens and record these underneath each column. When exchanging, children should write the digits above the relevant column. In most cases regrouping and exchanging will involve writing a 1 to represent a ten, however at times a different digit will need to be written where there has been exchange as shown in the example above.

Key Questions/Vocabulary

Addition, add, plus, more, more than
 Subtract, take away, minus, less than
 Difference between, inverse
 Equals, leaves, altogether
 Addend, sum, minuend, subtrahend
 Regrouping
 Exchange

**Know related
 subtraction facts for
 numbers within 20.**

Progression

- Children should begin using these methods where there is only one column that needs regrouping/exchanging.
- They should then progress onto calculations where multiple columns need regrouping/exchanges.
- They should then progress onto calculations featuring at least 2 4-digit numbers.
- They should be able to add numbers with up to 2 decimal places.
- They must be able to add 2 or more numbers including calculations whose addends have different numbers of digits.